Performance Analysis of Stochastic Fair Sharing Scheme for Link Sharing

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***Abstract*—We address the problem of the performance analysis of the stochastic fair sharing (SFS) algorithm for fair link sharing. The SFS scheme has been proposed to carry out a fair link sharing and fair sharing among virtual private networks. Depending upon the current utilization and provisioned capacities of the classes, the SFS admission control algorithm decides which sessions to accept and which to reject. In this letter, we undertake the performance evaluation of the SFS scheme analytically. We explore the tradeoff between fairness and the blocking probability by varying the trunk reservation parameter. The results show that the analytical performance measure agrees well with the simulation results.**

***Index Terms*—Link sharing, routing and resource allocation, stochastic fair sharing (SFS), virtual private networks (VPNs).**

I. Introduction

LINK-SHARING schemes have been proposed to allow the service providers to lease a part of their physical link to independent organizations (through their virtual private networks (VPNs) [2]). The complete sharing scheme delivers maximum possible bandwidth (BW) usage efficiency, while the complete partitioning scheme provides "fairness." In order to optimally use the BW capacity of the physical link and at the same time retain the fairness to the VPNs of varying session arrival rates, there were schemes [1], [3] proposed in the literature which gave priority to the underloaded VPNs. In the stochastic fair sharing (SFS) scheme proposed in [1], a certain amount of BW is reserved for a VPN of lower normalized BW usage before accepting a session belonging to a VPN of higher normalized BW usage. In SFS, the unused free capacity is *fairly* redistributed by resizing the capacity allocations depending upon the current usage of different VPNs sharing the link. On the contrary, in the scheme proposed in [3], the free capacity *cannot* be redistributed *fairly* over the overloaded classes using the technique of trunk reservation, although the underloaded classes are given priority over the overloaded classes while accepting the sessions. For example, in this scheme, a high session arrival rate may take the residual capacity of all classes.

The problem of BW allocation to a VPN and a typical IP service is significantly different, since the dynamism of these services are different with respect to their time scale of holding times. Typically, a VPN connection requires that a fixed BW is reserved for it for weeks or even months together, whereas a typical IP service has a holding time of just a few minutes. This requires a redefinition of the notion of "fairness"[1] as defined by Parekh and Gallager [4]. This has been done in [1], where the notion of fairness is also extended to the concept of BW reservations.

In general, while studying the performance of systems where very complex models are encountered, simulation techniques are successfully employed and sometimes preferred for analysis due to the intractability of such models. However, there is a certain need for analytical results to get deeper insight, reduce runtimes, handle very rare events, and optimize system performance, whenever possible. Toward this end, we consider the analytical performance evaluation of the SFS scheme which [1] lacks. In this letter, we analytically derive the blocking probability (in terms of the parameters of the SFS scheme) for sessions belonging to a class (say, a VPN). (For the analysis, we consider the discrete version of SFS, i.e., the session arrivals can request now for BWs which are discrete). It would analytically give a tradeoff between fairness and efficiency of BW usage. The paper is organized as follows. The SFS scheme is explained briefly in Section II. Section III discusses the analysis of the SFS scheme, where we arrive at the global balance equation which explains the dynamics of the SFS scheme. We then present the relevant simulation results. Section IV gives the simulation results pertaining to the fairness ratio and the blocking probability. Conclusions are presented in Section V.

II. SFS Scheme for Link Sharing

In this section, we describe the SFS scheme for the case of sharing in a single link. For more details, please refer to [1]. We consider a link of capacity C to be partitioned into N logical links (or classes) of provisioned capacities C_i, such that $\sum_{i=1}^{N} C_i \leq C$. We assume that real-time sessions arrive randomly according to a Poisson process. BW is reserved upon session arrivals and is released upon session completion. There is an admission control entity at the link which decides whether the link has adequate free capacity to accept the reservation requests of sessions. The session is said to be blocked if the session cannot be accepted.

Let r_i be the amount of capacity currently used by a logical link i. The normalized usage of logical link i is given by $n_i = r_i/C_i$. Consider the logical links being labeled in increasing order of their normalized usage. A new session of the ith class is accepted only if the free capacity after accepting the session is greater than or equal to the sum of the trunk reservation with lower normalized usage. Mathematically, a new session of logical link i, with BW request b_i is accepted if and only if $\sum_{j=1}^{N} r_j + b_i + \sum_{j<i} t_j \leq C$,

[1]In the special context of the above time scale factor.

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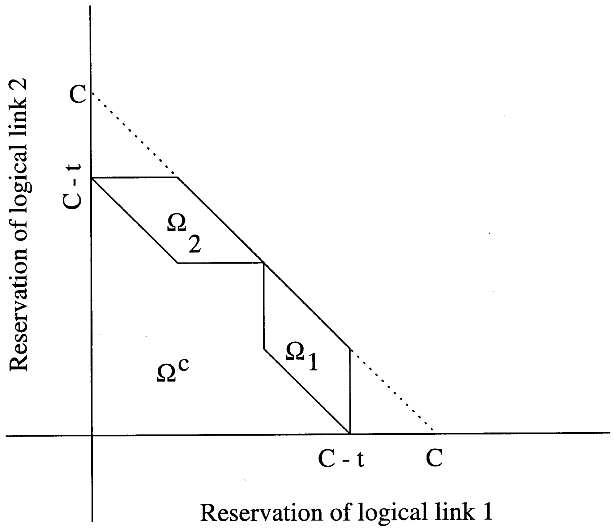

Fig. 1. State-space diagram for a two-dimensional SFS scheme for a link.

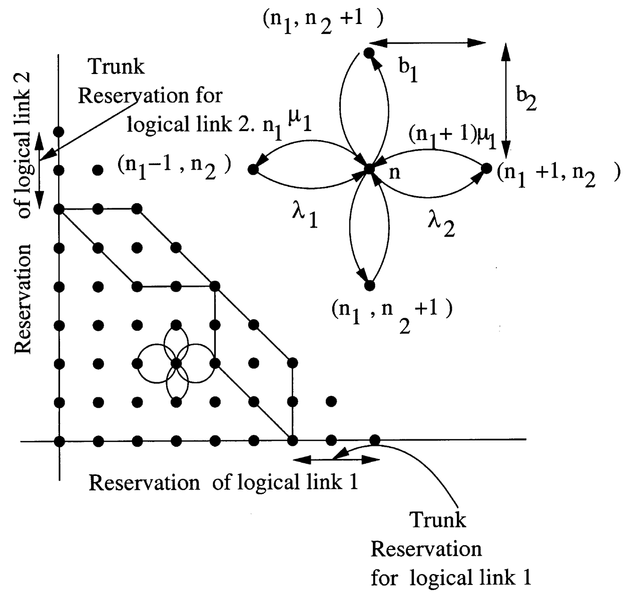

Fig. 2. State-space diagram for a link with $C = 8$ for a discrete analogous system.

where t_j is the *trunk reservation* for class j. The logical link with the lowest normalized usage is given the highest priority while accepting the sessions, and hence, sees a very low blocking probability. If the normalized usage n_i of a logical link i is close to its *fair share* denoted by f_i (described below), then it is not necessary to have a large value of trunk reservation for the logical link. Hence, the trunk reservation t_i is set to a static (maximum) trunk reservation parameter $\hat{t}_i$ when the difference between the fair share of a logical link and its current usage is large, and is set to this difference if the difference is less than its static trunk reservation. Formally, $t_i = \min[\hat{t}_i, f_i - r_i]$. The fair share of the logical link is the share it gets when the free capacity of logical links with lower normalized usage is shared by logical links with higher normalized usage. It is computed by redistributing the free capacity of logical links with lower normalized usage as follows: $f_i = (C_i/(\sum_{j=i}^{N} C_j))(C - \sum_{j=1}^{i-1}(r_j + t_j))$. The above expression is a natural generalization of the fairness criteria [4] used in packet schedulers.

We next use the state-space diagram to get a deeper insight into the behavior of the SFS call admission algorithm. Consider a physical link being shared by two "logical" links. The state of the (physical) link (as represented by any point in the state-space diagram) at any given time is represented by the current BW reservation of the two logical links. We assume that the trunk reservation and BW request for both the logical links are the same ($t_1 = t_2 = t$, $b_1 = b_2 = b$). The state-space diagram for a two-dimensional SFS system (two logical links sharing a physical link) is illustrated in Figs. 1 and 2. Fig. 1 illustrates the state-space diagram for the SFS scheme for the continuous case (session arrivals carry requests of bandwidth of continuous value). Fig. 2 illustrates the SFS for the discrete case. (Trunk reservation for both the users equals two). The X and Y axes represent the current reservation of the first and second logical link, respectively. The current state of the link can be represented by a point in the state-space diagram. When a new session on the first logical link is accepted, the system moves to the right, while upon acceptance of a new session on the second logical link, it moves up. When the sessions of the first or the second logical link complete, the system moves toward the left or downwards. As long as the total free capacity is greater than the trunk reservation, the system may move in any direction (assuming that the BW requirement of the sessions is small). This area is denoted by Ω^c in the figure. In the region denoted by Ω_2, the normalized utilization of logical link 2 is greater than that for the normalized utilization of logical link 1. Moreover, in this region, the total free capacity is less than the sum of the trunk reservation (for logical link 1) and the capacity requirement of the session belonging to logical link 2. Hence, the class 2 session arrivals are prohibited. Thus, in this region, the system can move left, right, or down, but not upwards. Similarly, in the region denoted by Ω_1, the system can move left, up, or down, but not toward the right. In this region, class 1 session arrivals are prohibited. We define, for our convenience, $\Omega \triangleq \Omega^c \bigcup_l \Omega_l$. Naturally, Ω forms the set of all allowable states.

III. Analysis of the SFS Scheme

In this section, we consider the session blocking probability in a logical link (say a VPN).

A. Model, Assumptions, and Notations

We assume that sessions of class k arrive according to a Poisson process with parameter λ_k, and have exponential holding times with mean $1/\mu_k$. The physical link capacity is C units. We use $\mathbf{n}$ to denote the random vector $\mathbf{n} \triangleq (\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_K)$, where $\mathbf{n}_i$ is the random variable denoting the number of type i sessions using the physical link, where K is the number of classes of traffic handled by the physical link. Denote the stationary probability $P(n)$ of the system in state $\mathbf{n} = n$, i.e., $P(n) \triangleq \Pr\{\mathbf{n} = n\}$. We use b_k to denote the BW requirement for the session arrival of the kth class and $\mathbf{b}$ to denote the vector $(b_1, b_2, \ldots, b_K)$. Define $\Gamma(i) \triangleq \{\mathbf{n}|\mathbf{n}.\mathbf{b} = i\}$, where the notation $\mathbf{n} \cdot \mathbf{b}$ is used to denote the sum $\sum_{k=1}^{K} \mathbf{n}_k b_k$. Let $q(i) \triangleq \Pr\{\mathbf{b}.\mathbf{n} = i\}$. Finally, we assume a symmetrical system, i.e., $t_i = t$ for $i = 1, 2, \ldots, K$. We need the following notation: $\mathbf{n}_i^+ = (n_1, \ldots, n_{i-1}, n_i + 1, n_{i+1}, \ldots, n_K)$, and

$\mathbf{n}_i^- = (n_1, \ldots, n_{i-1}, n_i - 1, n_{i+1}, \ldots, n_K)$. Define the functions $\gamma_i^+(\mathbf{n}) = \begin{cases} 1, & \sum_{j=1}^k n_j + b_i + \sum_{j \in S_i} t_j \text{ and } \mathbf{n} \in \Omega \\ 0, & \text{otherwise,} \end{cases}$ where $S_i = \{j | n_j < n_i\}$. $\gamma_i^-(\mathbf{n}) = \begin{cases} 1, & \mathbf{n}_i^- \in \Omega \text{ and } \mathbf{n} \in \Omega \\ 0, & \text{otherwise} \end{cases}$ $\alpha_i^-(\mathbf{n}) = \gamma_i^+(\mathbf{n}_i^-)$ and $\alpha_i^+(\mathbf{n}) = \gamma_i^-(\mathbf{n}_i^+)$. One can write the global balance equations for SFS, using the definitions above, as

$$\left[\sum_{i=1}^k \lambda_i \gamma_i^+(n) + \sum_{i=1}^k n_i \mu_i \gamma_i^-(n)\right] P(n) = \sum_{i=1}^k \lambda_i \alpha_i^-(n) P(n_i^-) + \sum_{i=1}^k (n_i + 1) \mu_i \alpha_i^+(n) P\left(n_i^+\right). \quad (1)$$

It is worth noting that the link occupancy constitutes an irreducible Markov process with the feasible region Ω as the state space.

B. Simulation Results

We carry out experiments to verify the global balance (1) using simulations. In addition, the above experiments also aim to study how the blocking probability varies with respect to trunk reservation parameter t. The SFS critically relies on the assumptions that: a) individual-session BW requirement is small as compared with the link capacity; b) most of the sessions have small holding time; and c) the session arrival process is not very bursty. These assumptions also seem reasonable in the scenario of many VPNs sharing a physical link. For instance, consider the transmission of an MPEG video which might be a session within a VPN service. The BW requirement of MPEG video streams is between 1–6 Mb/s, which is reasonably small compared with the link speeds of 155 Mb/s. Moreover, the session arrival process is not as bursty as the data arrival process. We therefore simulated using the most simplistic model as given below. Hence, we believe that the parameters used below in our simulation give a glimpse into the SFS performance *analytically*.

First, we consider an SFS system with the following parameters: $C = 8, K = 2, b_1 = 1, b_2 = 1, \lambda_1 = 3.0, \lambda_2 = 3.6, \mu_1 = 4.5$, and $\mu_2 = 4.0$. We numerically compute the blocking probability from (1) for various values of t. We carry out the simulations to validate the numerical computation of the blocking probability for the same values of t. Simulations are run for 1000 simulated seconds and are repeated a sufficient number of times with different seeds to get better estimates of the blocking probability. Our results are plotted in Figs. 3 and 4. It is evident from the figures that the numerical computation of blocking probability agrees with that of the simulated one. Moreover, one can see from the plot that when $t = 0$, the blocking probability is minimum. This is consistent with the intuition that when we have complete sharing ($t = 0$), the blocking probability should be minimum.

IV. Fairness of the SFS Scheme

In this section, we explore the fairness of the SFS scheme. For a fairness index, we use the following definition by Jain *et al.* [5]. Before defining the fairness index, we need the concept of equivalent capacity. Consider a single link with sessions (with requests of equal BW) arriving at Poisson rate. Session holding times are exponentially distributed. Then, the session blocking probability is given by the famous Erlang's formula [6]. Given

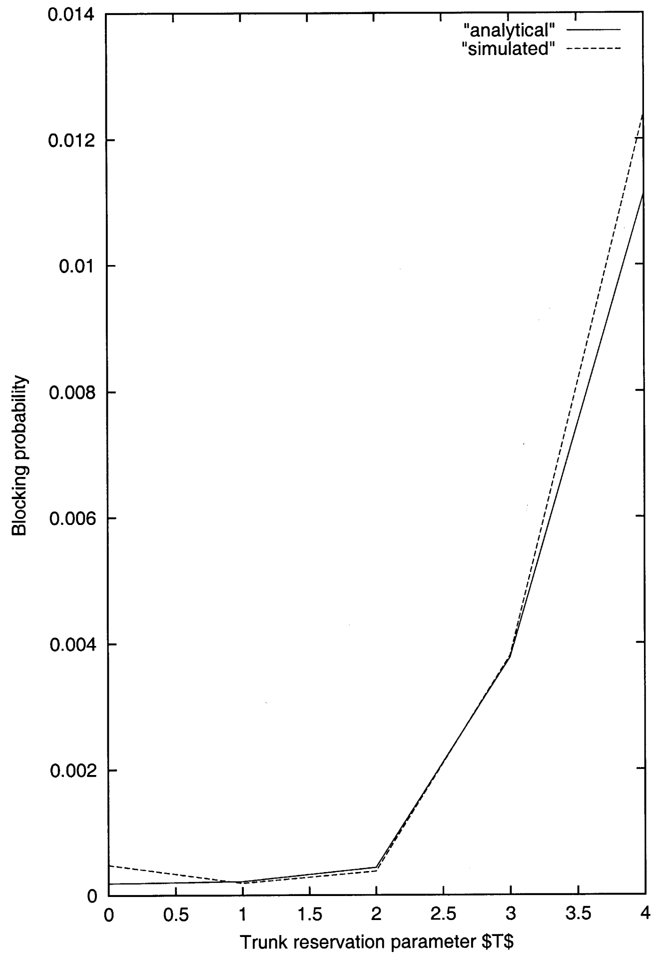

Fig. 3. Experimental verification of the theoretical evaluation of blocking probability for a class of type 1.

λ, the mean session arrival rate multiplied by the mean session holding time (λ also called "arrival rate" in Erlang's), the session blocking probability is given by

$$E(\lambda, C) = \frac{\frac{\lambda^C}{C!}}{\sum_{i=0}^{C} \frac{\lambda^i}{i!}} \quad (2)$$

where C is the physical link capacity (in units of number of sessions it can carry). Let Th_i and p_i be the mean throughput and blocking probability on logical link i. The mean arrival rate λ_i on the logical link is given by $\lambda_i = (Th_i/(1 - p_i))$. The equivalent capacity (C^e) of the logical link is defined as the capacity at which the arrival rate $(Th_i/(1 - p_i))$ results into a blocking probability of p_i.

$$C_i^e \triangleq \left\{ C | E\left(\frac{Th_i}{(1 - p_i)}, C\right) = p_i \right\}. \quad (3)$$

The *fairness ratio* is defined as the ratio of the equivalent capacity to the max-min fair capacity [7]. Now, the fairness index is defined as

$$I = \frac{\left(\sum_{i=1}^n f_i\right)^2}{\left(n\left(\sum_{i=1}^n f_i^2\right)\right)} \quad (4)$$

where f_i is the fairness ratio of the logical link i, and n is the number of logical links. Note that a fairness index of one implies perfect fairness, and a fairness index of zero implies gross unfairness.

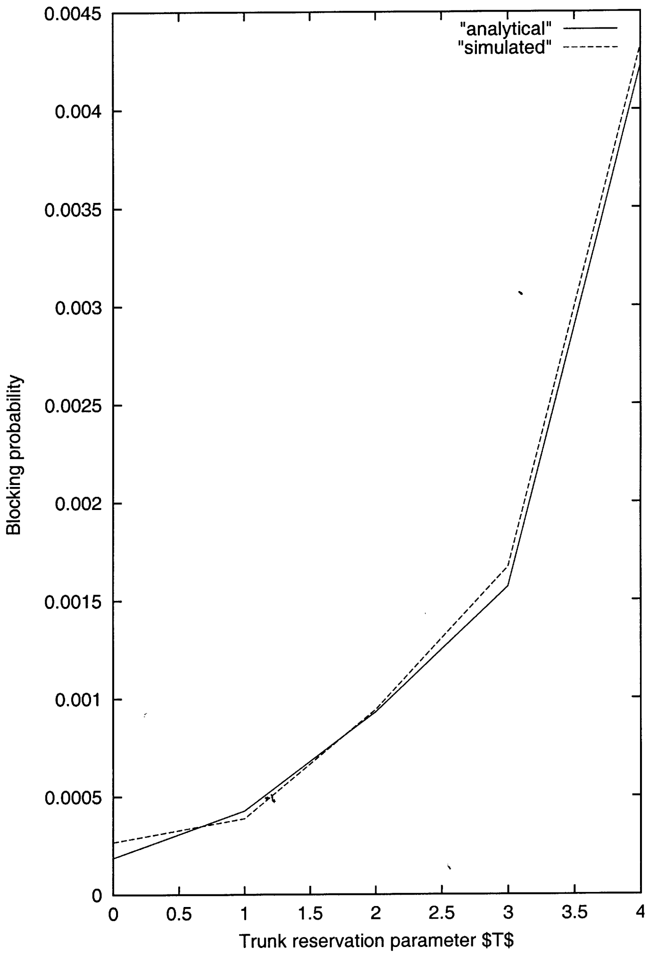

Fig. 4. Experimental verification of the theoretical evaluation of blocking probability for a class of type 2.

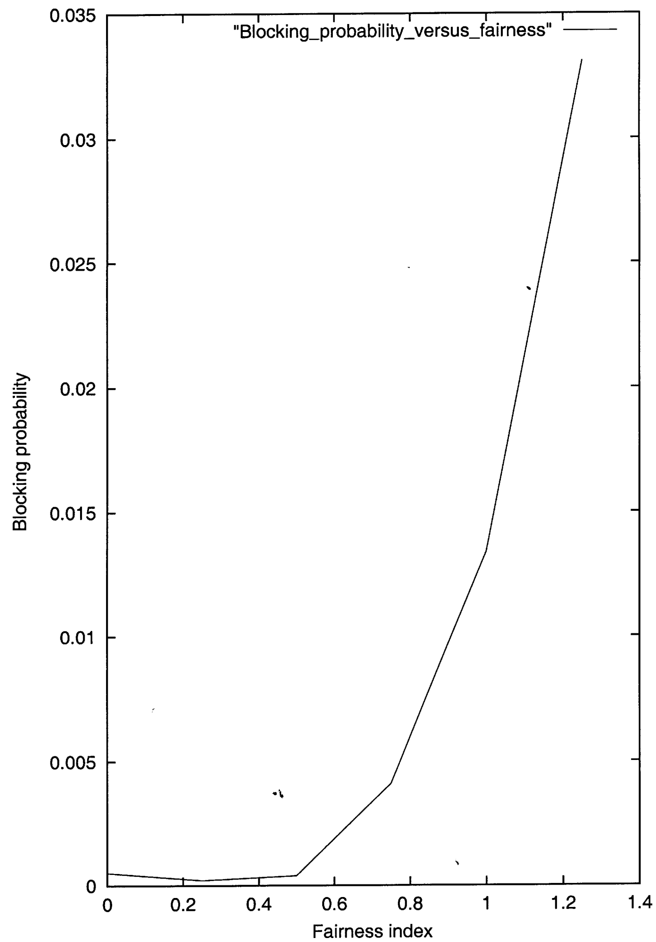

Fig. 5. Blocking probability versus fairness index.

In our study, we carried out the simulations to explore how the blocking probability varies with respect to the fairness ratio. We assume some arrival rates and service rates. Then, we assume a particular value for the trunk reservation parameter t. We compute p_i [from (1)] and $Th_i = (1 - p_i)\lambda_i$. Then, we find the equivalent capacity C_i^e given by (3). Next, we compute the max-min share. Finally, we compute the fairness ratio. Now, we change the trunk reservation parameter t to get another set of results. Note that when the trunk reservation t is varied, the blocking probability changes, but the ratio $(Th_i/(1 - p_i))$ remains unchanged.

Fig. 5 shows the blocking probability versus the fairness index. Note that the blocking probability is minimum when the fairness index is zero. We noted that zero fairness implies gross unfairness. This corresponds to complete sharing. Also note that the trunk reservation parameter is zero here. On the other hand, the maximum fairness (of one) denotes complete partitioning. The computation of equivalent capacity from Erlang's formula introduces a small error, since in our situation, session requests are of variable BW. This results in the fairness ratio being estimated greater than 1.0.

V. Conclusions

In this paper, we studied the analytical performance evaluation of the SFS scheme to carry out fair link sharing. We developed a fairly accurate model based on the Markovian model. The main performance measure is the session-blocking probability. We found that there was a good match between the blocking probability computed from the global balance equation and the simulations. Our work is significant in the context that the computation of blocking probabilities from the global balance equation, in terms of the parameters of the system *analytically*, will help us to understand the behavior of the continuous SFS scheme better.

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